The Stochastic Schrödinger Equations in selected physics problems

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Workshop of the *Espace de Structure Nucléaire Théorique*

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I. PROBLEMATIC

Stochastic methods have received much attention during the last decade in various fields of investigation. They grew rapidly in popularity with a large number of "industry standard" approaches for solving challenging global linear and non linear problems arising in physics. Their mathematical and numerical analysis is one of the most attractive research area in pure and applied probability, advanced stochastic engineering and computational physics and chemistry.
The "stochastic Schrödinger equations" are a part of the family of the stochastic partial differential equations which appear in numerous physics problems. Stochastic methods also stand the crossroad between different physics domains. Physical problems that were appearing as non-solvable can now being solved with stochastic methods and more specifically stochastic Schrödinger methods. In recent decades numerous examples have shown up in nuclear physics, condensed matter, ultracold atoms or quantum dots, to name few of them. One of the most successful theoretical result lies in the field of measure in quantum mechanics. An important and well established area of quantum optics is related to the theory of Markovian stochastic Schrödinger equations also known as quantum trajectory theory. Mathematically, this theory gave rise to stochastic Schrödinger equations, that is, pertubations of Schrödinger-type equations under the form of stochastic differential equations. Recently stochastic Schrödinger equations have also been developed for non-Markovian systems providing solutions for linear and non linear ones. These equations have proved to be a useful tool in the study of open quantum systems, i.e., quantum systems interacting with the surrounding environment.

However, while some accurate numerical methods exist for particular classes of stochastic Schrödinger equations, they are generally not extended to other domains. Physical and mathematical proofs, in order to justify the use of such equations, are far from being intuitive and evident. On the one hand, one can use non rigorous argument based on heuristic rules. On the other hand, in order to rigorously derive these models, one must use heavy and very abstract mathematical tools. In other words, physicist often encounter very specific difficulties associated to the use of non-linear stochastic Schrödinger equation.

In the special field of nuclear physics, a common feature is the stochastic nature of physical problems. Many situations are studied using phenomenological models that introduce adjusted parameters. Generally this adjustement is achieved to reproduce global properties of nuclei. Illustrations can be found in reaction models (optical model, statistical model,...) as well as in nuclear structure (phenomenological effective interactions as the Gogny force or

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the Skyrme ones, ...). Stochastic methods can be introduced in order to provide with a phenomenological theory of fluctuating phenomena. Even, under certain conditions random Hamiltonian evolutions based on a Gaussian white noise are investigated using stochastic processes.

This workshop will provide the opportunity to bring together and to promote interaction among researchers interested in the theoretical and applied aspects of stochastic algorithms in the field of stochastic Schrödinger equations. One of the goal of the workshop is to bring together experts in physical and mathematical aspects related to these equations.

II. GOALS OF THE WORKSHOP

In summary, the goals of the workshop are to

1. formulate problems in many-body physics in terms of stochastic Schrödinger equations.
2. provide a forum to discuss and share research and development experiences in the field of stochastic Schrödinger equations
3. discuss interactions between the different mathematical stochastic methods for partial differential equations applied in the fields of quantum mechanics.
4. promote and stimulate the exchanges of ideas between physicists and mathematicians.
5. cover applied and methodological aspects in stochastic Schrödinger equations.
6. to encourage and help young researchers (graduate students, postdocs) to do research in the subject.

III. USEFUL REFERENCES

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– N. Bouleau *Processus Stochastiques et applications* Hermann
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– D. Lacroix *Stochastic simulation of dissipation and non-Markovian effects in open quantum systems* Physical Review E, vol. 77, Issue 4, id. 041126
– X. Antoine :, *"Artificial boundary conditions for Schrödinger equations with general potentials and nonlinearities"* SAM Kolloquia 2010
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– H. Breuer and F. Petruccione, *The Theory of Open Quantum Systems* Oxford University Press, Oxford, 2002.
– A. Bulgac, *The Unitary Fermi Gas : From Monte Carlo to Density Functionals* arXiv :1008.3933
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– W. Gardiner and P. Zoller, *Quantum Noise* (Springer, Berlin, 2000).
– C. Hagendorf, C. Texier *Breaking supersymmetry in a one-dimensional random Hamiltonian* Journal of physics. A, Mathematical and theoretical 2008, vol. 41, no40

IV. LIST OF MAIN SPEAKERS

– C. Besse (laboratoire Paul Painlevé, Université de Lille)
– J. Bonnard (LPC Caen)
– A. Bulgac-(INT, Seattle USA)
– A. Debussche (ENS Cachan, Antenne de Bretagne)
– S. Descombes (Laboratoire J. Dieudonné, Nice)
– L. Di Menza (Laboratoire de Mathématiques de Reims)
– L. Diosi (Research Institute for Particle and Nuclear Physics-Hungary)

– J. Garnier (Laboratoire Pierre Louis Lions - Université Paris VII)
– M. Gregoratti (Politecnico di Milano)
– O. Juillet (LPC Caen)
– D. Lacroix (Ganil)
– R. Marty (Institut Elie Cartan Nancy)
– J. T. Stockburger-(Univ. Ulm, Germany)

V. SUPPORTS AND ANNOUCEMENT

The workshop will be supported by :

* **ESNT** (Espace de Structure Nucléaire Théorique) CEA/DSM/SPhT et SPhN - DAM Ile de France/SPN.
* **ANR Microwave** Microlocal analysis and numerical methods for wave ANR : http ://microwave.math.cnrs.fr/

The announcement will be done by :

* **ESNT** (Espace de Structure Nucléaire Théorique) : http ://irfu.cea.fr.
* **GdR MASCOT-NUM** Research Group on Stochastic Analysis Methods for COdes and NUMerical treatments : http ://www.gdr-mascotnum.fr.
* **SFdS** (French Society of Statistics's) "Reliability and Uncertainties" working group : http ://www.sfds.asso.fr
* **SMAI** Sociéte de Mathématiques Appliquées et Industrielles : http ://smai.emath.fr/

VI. PROGRAM

The proposed planning steps are :

1. An introduction to the Stochastic processes.
   An introductory course for people who have not previously studied Stochastic theory relating appropriate mathematical concepts to various physical systems ; domains : stochastic methods, stochastic differential equations...
2. Schrödinger Equation Applications.
   This part is dedicated to introduce the Schrödinger equation and its different forms. It covers numerical techniques associated to the resolution of the Schrödinger Equation and gives an overview of the applications
3. Introduction to the Stochastic Shrodinger equation.
   On Stochastic Partial Differential Equations. Topics will be selected from : Markovian and non-Markovian Quantum Stochastic Methods with application to Nuclear and Quantum Physics.
4. The nonlinear Stochastic Shrödinger equation.
   Devoted to the study of nonlinear stochastic Schrödinger equations.

Tuesday, December 6	Wednesday, December 7
Introduction to the Stochastic Processes	Introduction to the Stochastic Shrödinger Equation
09h15 Welcome X. Antoine D. Lacroix	09h00 L. Diosi
09h30 J. Garnier	10h00 A. Bulgac
10h45 **Coffee Break**	11h00 **Coffee Break**
11h15 A. Debussche	11h30 D. Lacroix
12h30 **Lunch**	12h30 **Lunch**
Schrödinger Equation Application	The nonlinear Stochastic Shrödinger Equation
14h00 S. Descombes	14h00 R. Marty
14h45 C. Besse	14h45 O. Juillet
15h30 **Break**	15h30 **Break**
16h00 L. Di Menza	16h00 M. Gregoratti
16h45 J.T Stockburger	16h45 J. Bonnard
	17h05 **round table**
17h30 **Closing**	17h30

VII. PRELIMINARY TITLES

C. Besse : *"Absorbing boundary conditions for Schrodinger equations"*
S. Descombes : *"Splitting schemes for Schrödinger equations"*
L. Di Menza : *"Some numerical methods for stochastic Schrödinger equations"*
R. Marty : *"Error estimates of a time-splitting scheme for the random nonlinear Schrodinger equation"*.
J. Stockburger : *"De-quantizing memory : non-Markovian dynamics made simple ?"*
A. Bulgac : *Towards an implementation of a real-time path-integral for interacting fermions*
D. Lacroix : *The nuclear many--body problem : an open quantum systems perspective*
M. Gregoratti : *Feedback control of a two-level atom and of its fluorescence light* **O. Juillet** *Stochastic wavefunction approach to Hubbard-like models J. Bonnard Phase-free quantum Monte-Carlo method for the nuclear shell-model*